SYMMETRY

DETERMINATION OF THE TYPE B STANDARD UNCERTAINTY

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Abstract. The type B standard uncertainty is determined when there is not enough measurement data to calculate the type A standard uncertainty.The value and the degrees of freedom of the type B standard uncertainty can often be calculated by the simple procedure given here. The degrees of freedom enables reliable calculation of a interval and level of confidence, even in a case of small sample size. Because of the essential importance of the interval and level of confidence, mentioned, or equivalent, calculations should be put in the same category with other calculations related to the standard uncertainty.

Keywords. Measurement, type B standard uncertainty, degrees of freedom, interval and level of confidence, Guide to the expression of uncertainty in measurement.

1 Introduction

The type B standard uncertainty is determined when there is not enough measurement data to calculate the type A standard uncertainty.

Outstanding, widely used, the Guide to the expression of uncertainty in measurement, GUM, lacking more harmonized approach to determination of the type B standard uncertainty. A value and degrees of freedom of that uncertainty can often be calculated by a simple procedure given here.

The GUM in a subclause 4.3.5 provides a useful method of calculating a value of the type B standard uncertainty, based on a evaluated interval and level of confidence.

The degrees of freedom enables reliable calculation of a interval and level of confidence, even in a case of small sample size. [Kostić] 6.17

The GUM recommends that the degrees of freedom should ALWAYS BE GIVEN with the type A standard uncertainty [GUM] 8, 4.2.6. And that with the type B standard uncertainty MAY BE GIVEN its the degrees of freedom, if it is deemed useful for intended users of the measurement result, as well as in a detailed report about a result determination [GUM] 8, 7.2.1, 7.2.7.

Based on the definition of the degrees of freedom given here and a few statements, also provided that the initial evaluation is in a specific format, immediately come to the method given here for calculating the degrees of freedom of the type B standard uncertainty.

There are no commonly accepted definition of the degrees of freedom. In a literature, they are often find the definitions like following.

> Degrees of freedom is a sample size, minus, number of population parameters that are evaluated from this sample [Kostić] 6.17; [Clapham] PP 216; [Eisenhauer].

Here is an obvious deficiency of the term "number of population parameters that are evaluated from this sample." This term shall not to include estimates that are not used for the evaluation for which we determine degrees of freedom, because that can not have an effect on that evaluation.

2 Degrees of freedom

Degrees of freedom, ν, of a evaluation determined from a sample of size *n*, is given with the equation (1). A *R* is number of STATISTICAL population parameters which is determined from the sample and UTILIZED TO PERFORM THIS EVALUATION. [Njegić] 5.6.; [GUM] G.3.3; [Eisenhauer]

$$\nu = n - R \tag{1}$$

Thus defined the degrees of freedom of the evaluation, gives a value that is parameter of different distributions. For example, a sample from a population with the normal distribution for which the equation (1) gives degrees of freedom ν, have the T-distribution with parameter "degrees of freedom" equal to ν. Also, the standard uncertainty for which (1) gives degrees of freedom ν, have the T-distribution with parameter "degrees of freedom" equal to ν.

3 Determination of the type B standard uncertainty

If for a variable with the normal distribution are known only a half-width of a interval of confidence, Δ, and a level of confidence, P, from the equation (2) we can calculate a standard uncertainty of that variable, u. A number k is coverage factor for a level of confidence, P, and for the normal distribution, so may be obtained from a corresponding table. [GUM] 4.3.5

$$u = \frac{\Delta}{k} \tag{2}$$

Like the previous one, the standard uncertainty can be calculated for variables with different distributions, taking an appropriate coverage factor. [GUM] G.3.2

The evaluated uncertainty u from (2), may be taken for the type B standard uncertainty. [GUM] 4.3.5

The uncertainty from (2) can be evaluated on basis of questioning a people involved in a measurement procedure [GUM] 4.3.5. The interviewee should make an evaluation of a value of a quantity in a following, or equivalent, format.

Out of total M values, N is in a range from a to b.

The degrees of freedom of the uncertainty u, is calculated in accordance to the previously given definition of the degrees of freedom. As uncertainty evaluation is based on totally M values, having previously calculated a level of confidence, P, using (1) we get:

$$\nu = M - 1. \tag{3}$$

Values for (2) is calculated from the interviewee initial evaluations, as follows. [GUM] 4.3.5

$$\Delta = \frac{|a - b|}{2} \tag{4}$$

$$P = \frac{N}{M} \tag{5}$$

It is then determined a coverage factor, k, for a level of confidence, P, for possible additional parameters, and for a evaluated distribution. Finally, it is calculated a standard uncertainty, u. [GUM] 4.3.5, G.3.2

By testing interviewee, it can be performed validation, or increase accuracy, of such evaluation of the standard uncertainty. On the basis of that test it may be determined a correction of evaluation, like a correction for a measuring instrument. A correction can be used as an index of accuracy of a evaluation, or for correction of evaluations in order to increase accuracy.

3.1 Example

It is evaluated that from 12 measurement results of lengths of one type element, a half results (which individually are not known) has the values in the range from 10.07 mm to 10.15 mm. It has been evaluated that the results is from a population with the normal distribution. Evaluation of the interval has a negligible systematic error.

Determine: a) a most frequent length of the element, b) a standard uncertainty of this length and c) a degrees of freedom of this uncertainty.

Solution

a) The most frequent length is a mode. Evaluated confidence limits are a = 10.07 mm and b = 10.15 mm. The results of measurement of a length have the T-distribution, so a value of mode, l, is in a midpoint of the interval a to b [GUM] 4.3.5:

$$l = \frac{a+b}{2} = \frac{10.07\text{ mm} + 10.15\text{ mm}}{2} = 10.11\,\text{mm}\,.$$

c) Evaluation of a standard uncertainty will be carried out on the basis of total M = 12 results, from there, by using (3) we are calculate the degrees of freedom of this uncertainty:

$\nu = M - 1 = 12 - 1 = 11.$

b) Based on the evaluated confidence limits, by using (4) we are calculate a half-width of the interval of confidence:

$$\Delta = \frac{|a-b|}{2} = \frac{|10.07\text{ mm} - 10.15\text{ mm}|}{2} = 0.04\text{ mm}\,.$$

Based on the total numbers of results, and evaluated number of results in the interval of confidence, N = 12 / 2 = 6, whereas using (5) we calculate a level of confidence:

$$P = \frac{N}{M} = \frac{6}{12} = 0.50\,.$$

For a evaluated level of confidence, P, and for degrees of freedom, ν, whereas from a corresponding table for the T-distribution, we obtain a coverage factor k = 0.698. For a length l, by using (2) we calculate a standard uncertainty:

$$u = \frac{\Delta}{k} = \frac{0.04\text{ mm}}{0.698} = 0.057\text{ mm}\,.$$

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