Cetvorozicno merenje induktivnosti

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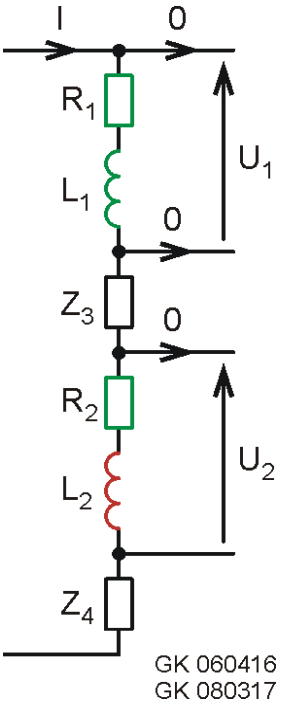


$f := 50.231 \cdot 10^{3}$ **Frekvencija struje.**

$R_1 := 0.01142$ **Otpornost (redna) referentne impedanse na** f**.**

$L_1 := 1.9372 \cdot 10^{-6}$ **Induktivnost (redna) referentne impedanse na** f**.**

$moduo\_U_1 := 0.997515$ **Izmeriti voltmetrom.**

$moduo\_U_2 := 0.0324615$ **Izmeriti voltmetrom.**

$R_2 := 17.965 \cdot 10^{-3}$ **Izmeriti ommetrom ako je skin efekat na** f **zanemariv.**

$\omega := 2 \cdot \pi \cdot f$

$Z_1 := R_1 + i \cdot \omega \cdot L_1$ $Z_1 = 11.42 \times 10^{-3} + 611.401i \times 10^{-3}$ $|Z_1| = 611.507656 \times 10^{-3}$

$moduo\_Z_2 := |Z_1| \cdot \frac{moduo\_U_2}{moduo\_U_1}$ $moduo\_Z_2 = 19.9 \times 10^{-3}$

$L_2 := \frac{\sqrt{moduo\_Z_2^{\,2} - R_2^{\,2}}}{\omega}$ $i \cdot \omega \cdot L_2 = 8.56i \times 10^{-3}$

$L_2 = 27.12044 \times 10^{-9}$ **Vrednost merene induktivnosti.**

Izvodjenje

$$\omega := 2 \cdot \pi \cdot f$$

$$Z_1 := R_1 + i \cdot \omega \cdot L_1$$

$$Z_2 := R_2 + i \cdot \omega \cdot L_2$$

$$\frac{U_1}{U_2} := \frac{I \cdot Z_1}{I \cdot Z_2}$$

$$\frac{U_1}{U_2} := \frac{Z_1}{Z_2}$$

$$\left|\frac{U_1}{U_2}\right| := \left|\frac{Z_1}{Z_2}\right|$$

$$\frac{|U_1|}{|U_2|} := \frac{|Z_1|}{|Z_2|}$$

$$|Z_2| := |Z_1| \cdot \frac{|U_2|}{|U_1|}$$

$$|Z_1| := \sqrt{R_1^{\,2} + (\omega \cdot L_1)^2}$$

$$|Z_2| := \sqrt{R_2^{\,2} + (\omega \cdot L_2)^2}$$

$$(|Z_2|)^2 := R_2^{\,2} + \omega^2 \cdot L_2^{\,2}$$

$$L_2^{\,2} := \frac{(|Z_2|)^2 - R_2^{\,2}}{\omega^2}$$

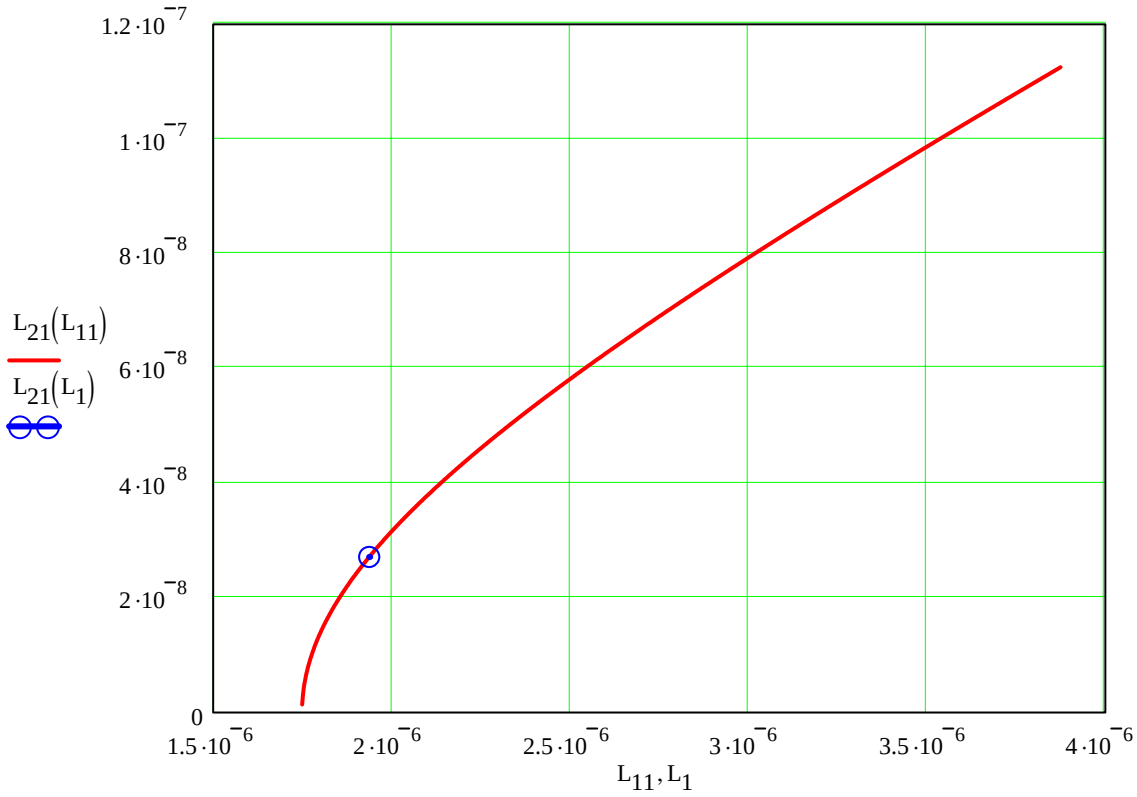
$$L_2 := \frac{\sqrt{(|Z_2|)^2 - R_2^{\,2}}}{\omega}$$

Greske

$$L_2 := \frac{\sqrt{\left[\left|R_1 + i\cdot(2\cdot\pi\cdot f)\cdot L_1\right|\cdot\frac{moduo\_U_2}{moduo\_U_1}\right]^2 - R_2^{\,2}}}{2\cdot\pi\cdot f}$$

$$L_{11} := 0.5\cdot L_1, 0.501\cdot L_1 .. 2\cdot L_1$$

$$L_{21}(L_{11}) := \frac{\sqrt{\left[\left|R_1 + i\cdot(2\cdot\pi\cdot f)\cdot L_{11}\right|\cdot\frac{moduo\_U_2}{moduo\_U_1}\right]^2 - R_2^{\,2}}}{2\cdot\pi\cdot f}$$



$$dL2\_po\_dL1 := \frac{2}{\left[\left(R_1^{\,2} + 4\cdot\pi^2\cdot f^{\,2}\cdot L_1^{\,2}\right)\cdot\frac{moduo\_U_2^{\,2}}{moduo\_U_1^{\,2}} - R_2^{\,2}\right]^{\frac{1}{2}}}\cdot\pi\cdot f\cdot L_1\cdot\frac{moduo\_U_2^{\,2}}{moduo\_U_1^{\,2}}$$

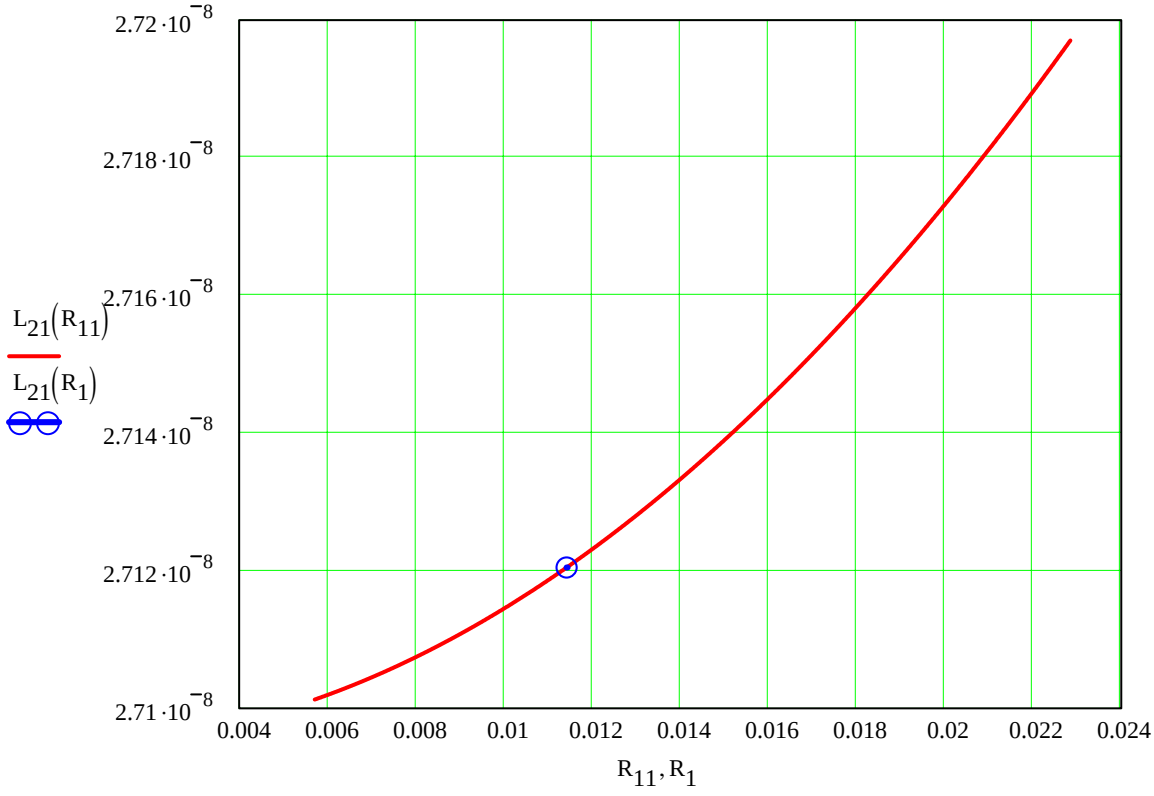
$dL2\_po\_dL1 = 0.076$

$$100\cdot\frac{dL2\_po\_dL1}{L_2}\cdot\frac{1}{100}\cdot L_1 = 5.403$$

promena L_2 u % za 1 % promene L_1

$R_{11} := 0.5 \cdot R_1, 0.501 \cdot R_1 .. 2 \cdot R_1$

$$L_{21}(R_{11}) := \frac{\sqrt{\left[\left|R_{11} + i \cdot (2 \cdot \pi \cdot f) \cdot L_1\right| \cdot \frac{moduo\_U_2}{moduo\_U_1}\right]^2 - R_2^{\,2}}}{2 \cdot \pi \cdot f}$$



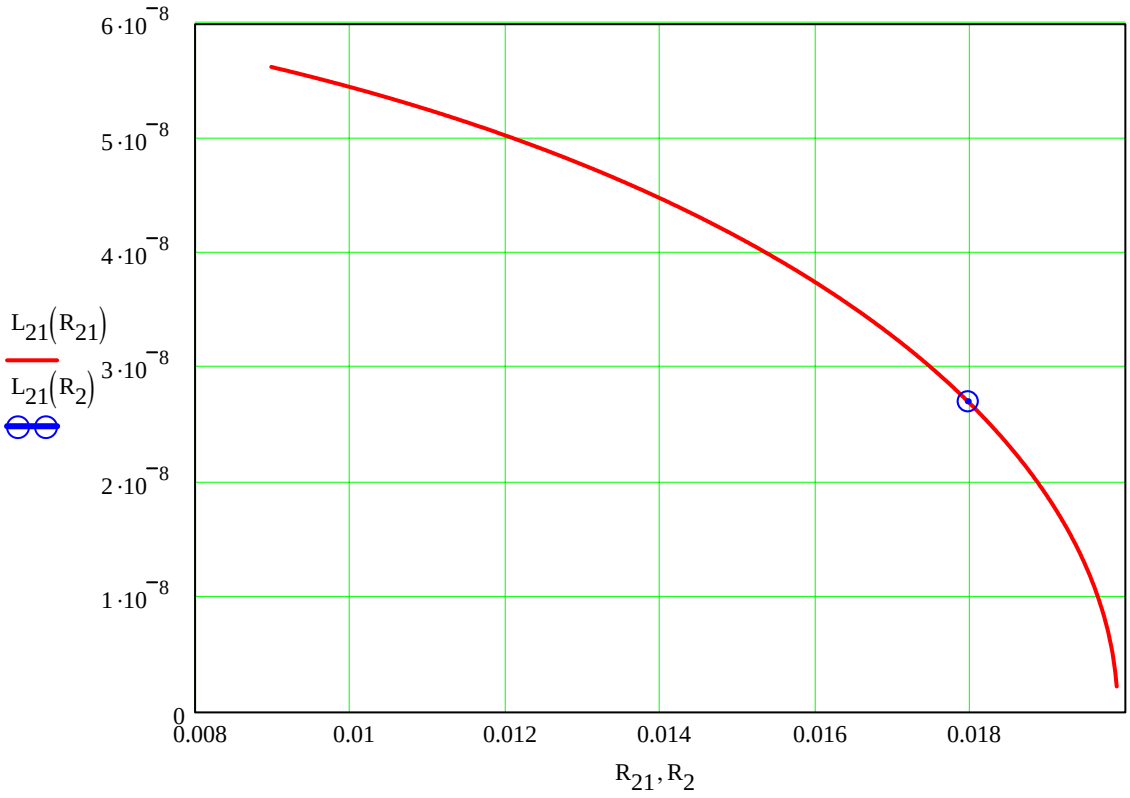
$$dL2\_po\_dR1 := \frac{1}{2 \cdot \left[\left(R_1^{\,2} + 4 \cdot \pi^2 \cdot f^2 \cdot L_1^{\,2}\right) \cdot \frac{moduo\_U_2^{\,2}}{moduo\_U_1^{\,2}} - R_2^{\,2}\right]^{\frac{1}{2}} \cdot \pi \cdot f} \cdot R_1 \cdot \frac{moduo\_U_2^{\,2}}{moduo\_U_1^{\,2}}$$

$dL2\_po\_dR1 = 4.477 \times 10^{-9}$

$100 \cdot \frac{dL2\_po\_dR1}{L_2} \cdot \frac{1}{100} \cdot R_1 = 1.885 \times 10^{-3}$ promena L_2 u % za 1 % promene R_1

$$R_{21} := 0.5 \cdot R_2, 0.501 \cdot R_2 .. 2 \cdot R_2$$

$$L_{21}(R_{21}) := \frac{\sqrt{\left[\left|R_1 + i \cdot (2 \cdot \pi \cdot f) \cdot L_1\right| \cdot \frac{moduo\_U_2}{moduo\_U_1}\right]^2 - R_{21}^{\ 2}}}{2 \cdot \pi \cdot f}$$



$$dL2\_po\_dR2 := \frac{-1}{2 \cdot \left[\left(R_1^{\ 2} + 4 \cdot \pi^2 \cdot f^{\ 2} \cdot L_1^{\ 2}\right) \cdot \frac{moduo\_U_2^{\ 2}}{moduo\_U_1^{\ 2}} - R_2^{\ 2}\right]^{\frac{1}{2}} \cdot \pi \cdot f} \cdot R_2$$

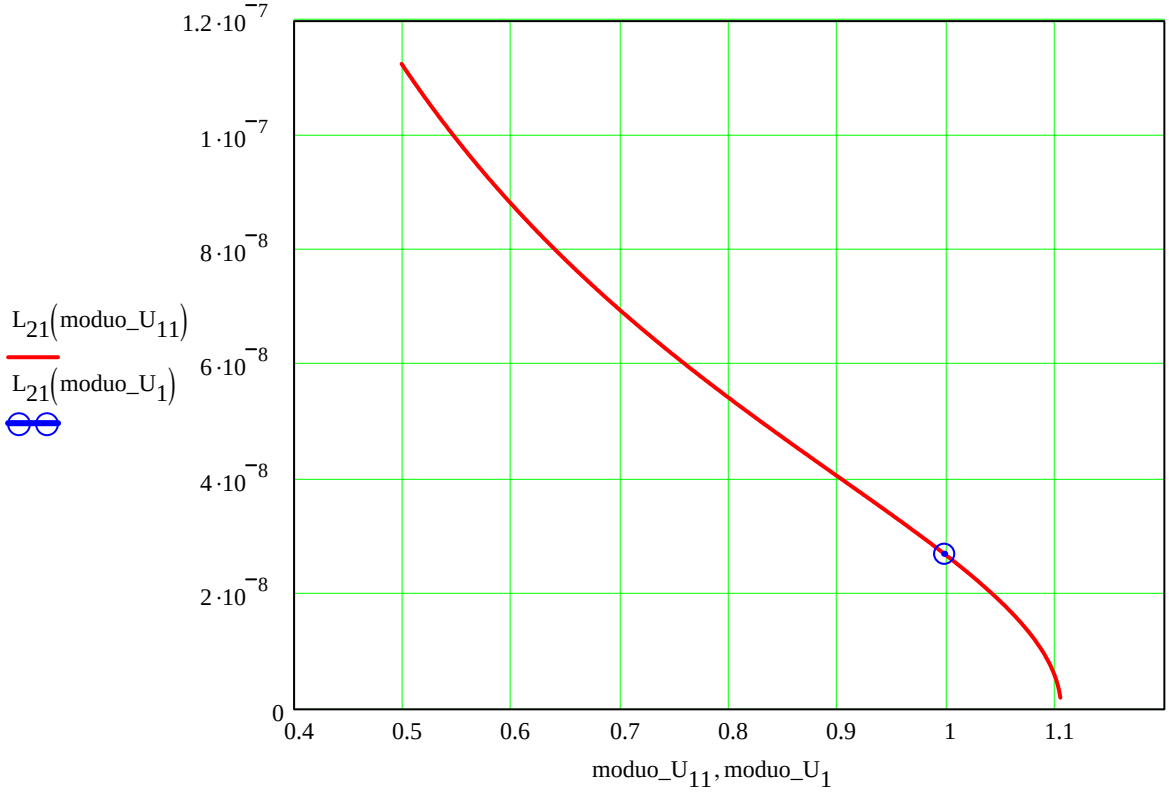
$$dL2\_po\_dR2 = -6.65 \times 10^{-6}$$

$$100 \cdot \frac{dL2\_po\_dR2}{L_2} \cdot \frac{1}{100} \cdot R_2 = -4.405$$

promena L_2 u % za 1 % promene R_2

$$moduo\_U_{11} := 0.5 \cdot moduo\_U_1, 0.501 \cdot moduo\_U_1 .. 2 \cdot moduo\_U_1$$

$$L_{21}(moduo\_U_{11}) := \frac{\sqrt{\left[\left|R_1 + i \cdot (2 \cdot \pi \cdot f) \cdot L_1\right| \cdot \frac{moduo\_U_2}{moduo\_U_{11}}\right]^2 - R_2^{\,2}}}{2 \cdot \pi \cdot f}$$



$$dL2\_po\_dmoduo\_U1 := \frac{-1}{2 \cdot \left[\left(R_1^{\,2} + 4 \cdot \pi^2 \cdot f^{\,2} \cdot L_1^{\,2}\right) \cdot \frac{moduo\_U_2^{\,2}}{moduo\_U_1^{\,2}} - R_2^{\,2}\right]^{\frac{1}{2}} \cdot \pi \cdot f} \cdot \left(R_1^{\,2} + 4 \cdot \pi^2 \cdot f^{\,2} \cdot L_1^{\,2}\right) \cdot \frac{moduo\_U_2^{\,2}}{moduo\_U_1^{\,3}}$$

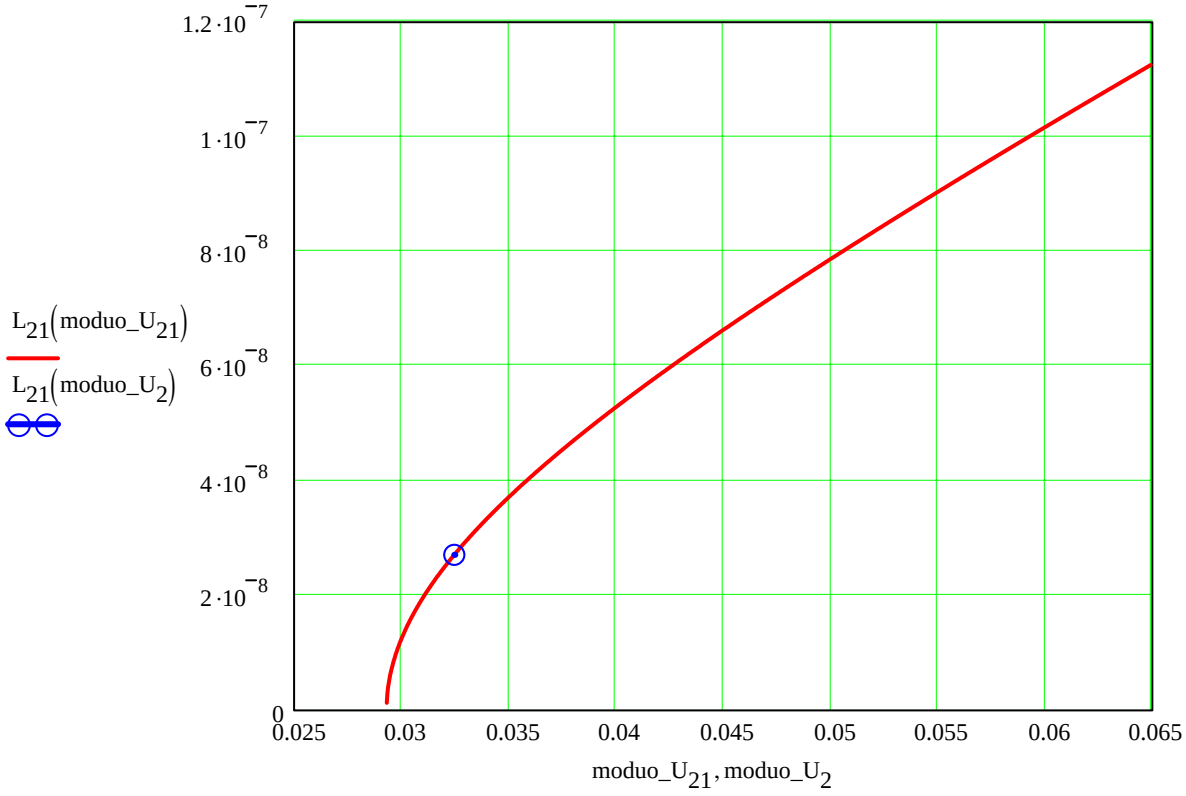
$$dL2\_po\_dmoduo\_U1 = -1.47 \times 10^{-7}$$

$$100 \cdot \frac{dL2\_po\_dmoduo\_U1}{L_2} \cdot \frac{1}{100} \cdot moduo\_U_1 = -5.405$$

promena L_2 u % za 1 % promene $moduo\_U_1$

$$moduo\_U_{21} := 0.5 \cdot moduo\_U_2, 0.501 \cdot moduo\_U_2 .. 2 \cdot moduo\_U_2$$

$$L_{21}(moduo\_U_{21}) := \frac{\sqrt{\left[\left|R_1 + i \cdot (2 \cdot \pi \cdot f) \cdot L_1\right| \cdot \frac{moduo\_U_{21}}{moduo\_U_1}\right]^2 - R_2^{\,2}}}{2 \cdot \pi \cdot f}$$



$$dL2\_po\_dmoduo\_U2 := \frac{1}{2 \cdot \left[\left(R_1^{\,2} + 4 \cdot \pi^2 \cdot f^{\,2} \cdot L_1^{\,2}\right) \cdot \frac{moduo\_U_2^{\,2}}{moduo\_U_1^{\,2}} - R_2^{\,2}\right]^{\frac{1}{2}} \cdot \pi \cdot f} \cdot \left(R_1^{\,2} + 4 \cdot \pi^2 \cdot f^{\,2} \cdot L_1^{\,2}\right) \cdot \frac{moduo\_U_2}{moduo\_U_1^{\,2}}$$

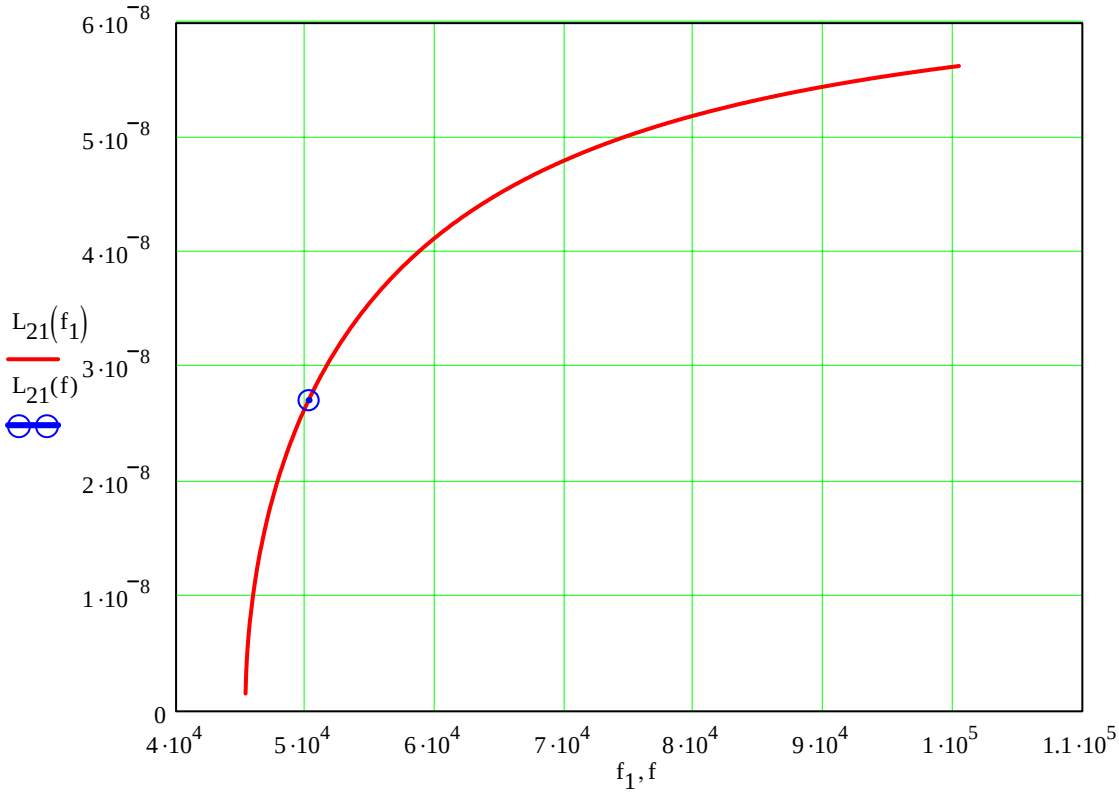
$$dL2\_po\_dmoduo\_U2 = 4.516 \times 10^{-6}$$

$$100 \cdot \frac{dL2\_po\_dmoduo\_U2}{L_2} \cdot \frac{1}{100} \cdot moduo\_U_2 = 5.405$$

promena L_2 u % za 1 % promene $moduo\_U_2$

$f_1 := 0.5 \cdot f, 0.501 \cdot f .. 2 \cdot f$

$$L_{21}(f_1) := \frac{\sqrt{\left[\left|R_1 + i \cdot (2 \cdot \pi \cdot f_1) \cdot L_1\right| \cdot \frac{moduo\_U_2}{moduo\_U_1}\right]^2 - R_2^{\,2}}}{2 \cdot \pi \cdot f_1}$$



$$dL2\_po\_df := \frac{2}{\left[\left(R_1^{\,2} + 4 \cdot \pi^2 \cdot f^{\,2} \cdot L_1^{\,2}\right) \cdot \frac{moduo\_U_2^{\,2}}{moduo\_U_1^{\,2}} - R_2^{\,2}\right]^{\frac{1}{2}}} \cdot \pi \cdot L_1^{\,2} \cdot \frac{moduo\_U_2^{\,2}}{moduo\_U_1^{\,2}} \ldots$$

$$+ -\frac{1}{2} \cdot \frac{\left[\left(R_1^{\,2} + 4 \cdot \pi^2 \cdot f^{\,2} \cdot L_1^{\,2}\right) \cdot \frac{moduo\_U_2^{\,2}}{moduo\_U_1^{\,2}} - R_2^{\,2}\right]^{\frac{1}{2}}}{\pi \cdot f^{\,2}}$$

$dL2\_po\_df = 2.377 \times 10^{-12}$

$100 \cdot \frac{dL2\_po\_df}{L_2} \cdot \frac{1}{100} \cdot f = 4.403$ promena L_2 u % za 1 % promene f

Kombinovana standardna nesigurnost izmerene vrednosti L_2

$u_{R\_L1} := 0.33$ [%]

$u_{R\_R1} := 1.0$ [%]

$u_{R\_R2} := 0.5$ [%]

$u_{R\_moduo\_U1} := 0.5$ [%]

$u_{R\_moduo\_U2} := 0.5$ [%]

$u_{R\_f} := 0.0$ [%]

$$V_{L1} := \left(\frac{u_{R\_L1}}{100}\cdot L_1\right)^2$$

$$V_{R1} := \left(\frac{u_{R\_R1}}{100}\cdot R_1\right)^2$$

$$V_{R2} := \left(\frac{u_{R\_R2}}{100}\cdot R_2\right)^2$$

$$V_{moduo\_U1} := \left(\frac{u_{R\_moduo\_U1}}{100}\cdot moduo\_U_1\right)^2$$

$$V_{moduo\_U2} := \left(\frac{u_{R\_moduo\_U2}}{100}\cdot moduo\_U_2\right)^2$$

$$V_f := \left(\frac{u_{R\_f}}{100}\cdot f\right)^2$$

$$V_{C\_L2} := dL2\_po\_dL1^2\cdot V_{L1} + dL2\_po\_dR1^2\cdot V_{R1} + dL2\_po\_dR2^2\cdot V_{R2} + dL2\_po\_dmoduo\_U1^2\cdot V_{moduo\_U1} \ldots + dL2\_po\_dmoduo\_U2^2\cdot V_{moduo\_U2} + dL2\_po\_df^2\cdot V_f$$

$$u_{C\_L2} := \sqrt{V_{C\_L2}}$$

$$u_{C\_L2} = 1.29\times 10^{-9}$$

$$u_{RC\_L2} := 100\cdot\frac{u_{C\_L2}}{L_2}$$

$$u_{RC\_L2} = 4.758 \quad [\%]$$

$$3\cdot u_{RC\_L2} = 14.274 \quad [\%]$$

$$L_2 = 27.12\times 10^{-9}$$

GK 060413, 080207, 080214, 080317